

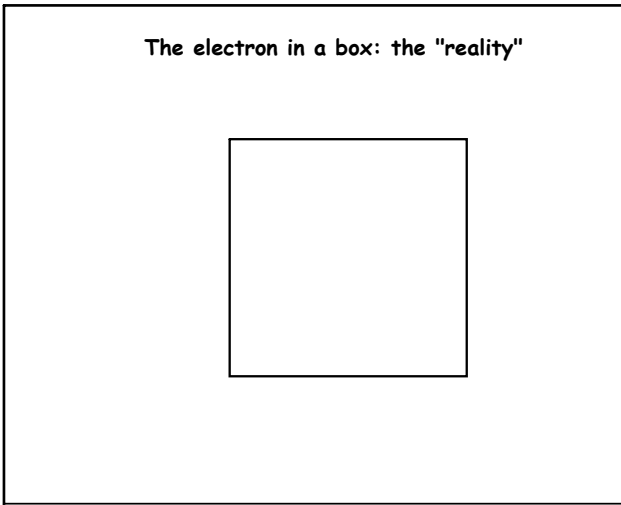
Probability and Quantum Mechanics  
"CONSERVATION OF PROBABILITY"

Probability and Quantum Mechanics  
The "Monte Hall" Problem

DOOR #1      DOOR #2      DOOR #3

**The Electron Cloud**  
Probability Functions: Defining and Using Them

The electron in a box



**Probability Conservation Equation**

Start from the probability and differentiate with respect to time.

$$\frac{\partial P(x, t)}{\partial t} = \frac{\partial}{\partial t} (\psi^*(x, t)\psi(x, t)) = \left[ \frac{\partial \psi^*}{\partial t} \psi - \psi^* \frac{\partial \psi}{\partial t} \right]$$

Use the Schrödinger Equation

$$\frac{-\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V(x)\psi = i\hbar \frac{\partial \psi}{\partial t}$$

and its complex conjugate

$$\frac{-\hbar^2}{2m} \frac{\partial^2 \psi^*}{\partial x^2} + V(x)\psi^* = -i\hbar \frac{\partial \psi^*}{\partial t}$$

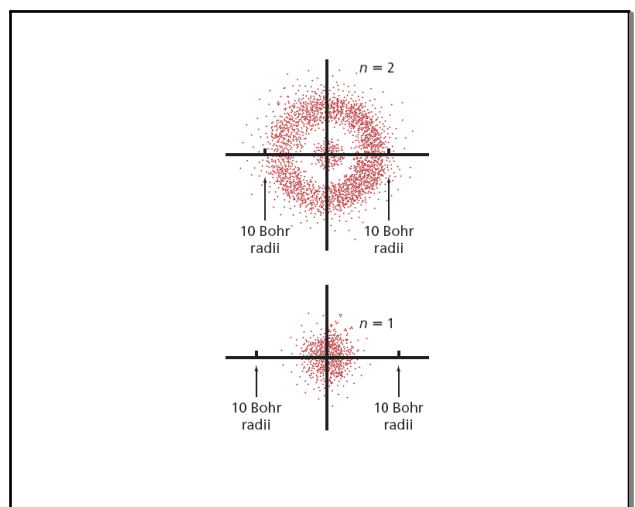
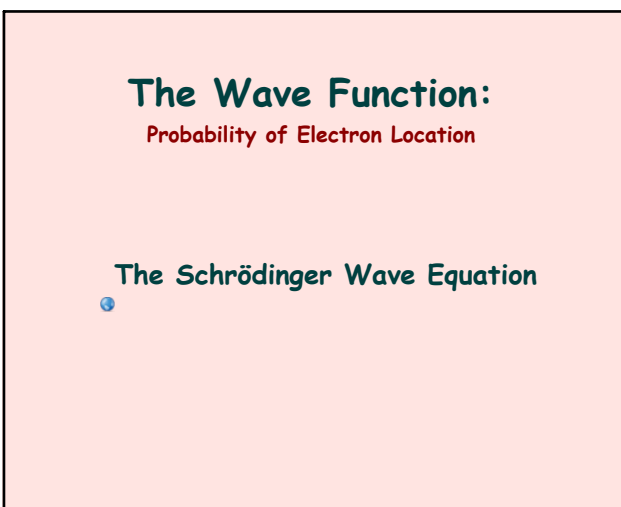
(We assume  $V(x)$  is real. Imaginary potentials do cause probability not to be conserved.)

Now we need to plug those equations in.

$$\begin{aligned} \frac{\partial P(x, t)}{\partial t} &= \frac{1}{i\hbar} \left[ \frac{\hbar^2}{2m} \frac{\partial^2 \psi^*}{\partial x^2} \psi - V(x)\psi^* \psi + \frac{-\hbar^2}{2m} \psi^* \frac{\partial^2 \psi}{\partial x^2} + V(x)\psi^* \psi \right] \\ &= \frac{1}{i\hbar} \frac{\hbar^2}{2m} \left[ \frac{\partial^2 \psi^*}{\partial x^2} \psi - \psi^* \frac{\partial^2 \psi}{\partial x^2} \right] = \frac{\hbar}{2mi} \frac{\partial}{\partial x} \left[ \frac{\partial \psi^*}{\partial x} \psi - \psi^* \frac{\partial \psi}{\partial x} \right] \end{aligned}$$

This is the usual conservation equation  $j(x, t)$  if is identified as the probability current.

$$\begin{aligned} \frac{\partial P(x, t)}{\partial t} + \frac{\partial j(x, t)}{\partial x} &= 0 \\ j(x, t) &= \frac{\hbar}{2mi} \left[ \psi^* \frac{\partial \psi}{\partial x} - \frac{\partial \psi^*}{\partial x} \psi \right] \end{aligned}$$



Why do we need "orbitals" to describe the location of electrons? Because electrons are constantly in motion!

The propeller example:



The propeller at rest



The propeller in motion

## The Heisenberg Uncertainty Principle

### Example 28-1, pg 792

An electron moves in a straight line with a constant speed  $1.10 \times 10^6$  m/s which has been measured to a precision of 0.10 percent. What is the maximum precision with which its position could be simultaneously measured?

### Example 28-2, pg 792

What is the uncertainty in position imposed by the uncertainty principle on a 150 g baseball thrown at  $93 \text{ mph} \pm 2 \text{ mph}$ ? ( $42 \text{ m/s} \pm 1 \text{ m/s}$ )